Statistics

Lecture 7

Feb 19-8:47 AM

SG 16

Binomial Prob. dist.

1) n independent events

2) Each event has only two outcomes

$P(\text{Success}) = p \qquad P(\text{Failure}) = q$

$$p + q = 1$$

$$q = 1 - p$$

p & q remain unchanged for all events.

3) $x \rightarrow$ # of Successes , $n - x \rightarrow$ # of failures

$$\boxed{P(x) = \boxed{{}_nC_x} \cdot p^x \cdot q^{n-x}}$$

↳ gives # of ways we can have x successes.

Apr 10-8:07 AM

$${}_nC_x = \frac{n!}{x!\cdot(n-x)!}$$

$${}_5C_2 = \frac{5!}{2!\cdot(5-2)!} = \frac{5!}{2!\cdot 3!} = \frac{5\cdot\overset{2}{\cancel{4}}\cdot\cancel{3\cdot2\cdot1}}{\cancel{2}\cdot1\cdot\cancel{3\cdot2\cdot1}} = \boxed{10}$$

5 trials, there are 10 ways to have 2 Successes,

Using TI:

5 [Math] → PRB ↓ [$_nC_r$] 2 [enter] $\quad {}_5C_2 = 10$ ✓

Apr 10-8:13 AM

$${}_{10}C_4 = \frac{10!}{4!\cdot(10-4)!}$$

$$= \frac{10!}{4!\cdot 6!} = \frac{10\cdot\overset{3}{\cancel{9}}\cdot\cancel{8}\cdot7\cdot\cancel{6!}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot1\cdot\cancel{6!}} = \boxed{210}$$

In 10 trials, there are 210 ways to have 4 Successes.

$$\boxed{{}_{10}C_4 = 210}$$

Using TI

10 [Math] → PRB ↓ [$_nC_r$] 4 [Enter]

Ca Lotto

choose 5 numbers in any order from 1 to 50.

How many ways can we do that?

$${}_{50}C_5 = 2{,}118{,}760$$

Apr 10-8:17 AM

Consider a binomial prob. dist. with
n = 5 and p = .6

1) $q = 1 - p$
$= 1 - .6 = \boxed{.4}$

2) $np = 5(.6)$
$= \boxed{3}$

3) npq
$= 5(.6)(.4)$
$= \boxed{1.2}$

4) $P(\underbrace{\text{exactly 2 Successes}})$

$$= P(x = 2) = {}_{\underset{n}{5}}C_{\underset{x}{2}} \cdot (\underset{p}{.6})^{\overset{x}{2}} \cdot (\underset{q}{.4})^{\overset{n-x}{3}}$$

$$= 10 \cdot (.6)^2 \cdot (.4)^3 = .2304$$
$$\approx \boxed{.230}$$

10 [×] .6 [^] 2 [×] .4 [^] 3 [Enter]

Using TI:

[2nd] [VARS] [↓] [binompdf(]

$P(x=2) = \text{binompdf}(5, .6, 2)$
$= \boxed{.230}$

Trials: 5
P: .6
x value: 2
[Paste] [Enter]

Apr 10-8:24 AM

I flip a fair coin 16 times and success is to land tails.

1) $n = 16$

2) $p = .5$

3) $q = .5$

4) np
$= 16 \cdot (.5) = \boxed{8}$

5) npq
$= 16(.5)(.5) = \boxed{4}$

6) $\sqrt{npq}$
$= \sqrt{4} = \boxed{2}$

7) $P(\underbrace{\text{get exactly 10 tails}})$

$$P(x = 10) = {}_{\underset{n}{16}}C_{\underset{x}{10}} \cdot (\underset{p}{.5})^{\overset{x}{10}} \cdot (\underset{q}{.5})^{\overset{n-x}{6}}$$

$$= 8008\,(.5)^{10} \cdot (.5)^6 \approx \boxed{.122}$$

Using TI

$$P(x=10) = \text{binompdf}(\overset{n}{16}, \overset{p}{.5}, \overset{x}{10}) \approx \boxed{.122}$$

Apr 10-8:33 AM

You are taking a test of multiple-choice with 20 questions. $n=20$

Each question has 4 choices with only one correct choice. $P(\text{Correct}) = \frac{1}{4} = .25$ $P(\overline{\text{correct}}) = \frac{3}{4} = .75$

You are making random guesses.

1) $n = 20$ 2) $p = .25$ 3) $q = .75$

4) $np = 20(.25) = \boxed{5}$

5) $npq = 20(.25)(.75) = \boxed{3.75}$

6) $\sqrt{npq} = \sqrt{3.75} \approx \boxed{2}$

7) $P(\text{guess } \underbrace{\text{exactly } 8} \text{ correct answers})$

$$P(x=8) = \text{binompdf}(\overset{n}{20}, \overset{p}{.25}, \overset{x}{8}) = \boxed{.061}$$

8) $P(\text{guess } \underbrace{\text{at most } 8} \text{ correct answers})$

$$P(x \le 8) = P(x=8) + P(x=7) + P(x=6) + \cdots + P(x=0)$$

$$= \text{binomcdf}(20, .25, 8) \approx \boxed{.959}$$

Apr 10-8:41 AM

We randomly select 40 newborn babies.

Success is to have a girl.

1) $n = 40$ 2) $p = .5$ 3) $q = .5$

4) $np = 40(.5) = \boxed{20}$

5) $npq = 40(.5)(.5) = \boxed{10}$

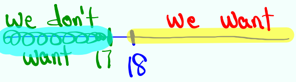
6) $\sqrt{npq} = \sqrt{10} \approx \boxed{3}$

7) $P(\underbrace{\text{exactly } 18} \text{ girls})$

$$P(x = 18) = \text{binompdf}(40, .5, 18) = \boxed{.103}$$

8) $P(\underbrace{\text{at least } 18} \text{ girls})$

$$P(x \ge 18) = P(x=18) + P(x=19) + P(x=20) + \cdots + P(x=40)$$



$P(x \ge 18) =$

$1 - P(x \le 17)$

Total Prob.

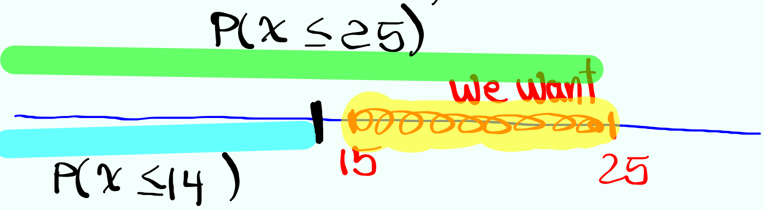
$= 1 - \text{binomcdf}(40, .5, 17)$

$= \boxed{.785}$

Apr 10-8:53 AM

9) P(we have $\underbrace{\text{between 15 and 25, inclusive,}}$ girls)

$$P(15 \le x \le 25) = P(x \le 25) - P(x \le 14)$$



= binomcdf(40, .5, 25) − binomcdf(40, .5, 14)

$= \boxed{.919} \approx 92\%$

Apr 10-9:03 AM

Consider a binomial prob. dist with
n = 400 & P = .8.

1) $q = 1 - p = \boxed{.2}$

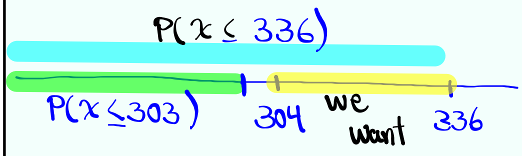
2) $np = 400(.8) = \boxed{320}$

3) $npq = 400(.8)(.2) = \boxed{64}$

4) $\sqrt{npq} = \sqrt{64} = \boxed{8}$

Let x be # of Successes

5) $P(304 \le x \le 336) = P(x \le 336) - P(x \le 303)$



$= \boxed{.961}$

= binomcdf(400, .8, 336) − binomcdf(400, .8, 303)

Apr 10-9:09 AM

Mean $\mu = np$

Variance $\sigma^2 = npq$

Standard Deviation $\sigma = \sqrt{\sigma^2}$

Binomial
Prob.
Dist.

Consider a binomial Prob. dist with $n = 120$ and $P = .4$.

$q = 1 - P = \boxed{.6}$

$\mu = np = 120(.4) = \boxed{48}$

$\sigma^2 = npq = 120(.4)(.6) = \boxed{28.8}$

$\sigma = \sqrt{\sigma^2} = \sqrt{28.8} = 5.367 \approx \boxed{5}$

68% Range $\mu \pm \sigma = 48 \pm 5 \rightarrow \boxed{43 \text{ to } 53}$

Let x be # of Successes

$P(43 \leq x \leq 53) = \text{binomcdf}(120, .4, 53) - \text{binomcdf}(120, .4, 42)$

$\approx \boxed{.695} \approx 69.5\%$

Apr 10-9:33 AM

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80 students were randomly Selected.

75% chance that any Student has a job.

1) $n = 80$

2) $p = .75$

3) $q = .25$

4) $\mu = np = 80(.75) = \boxed{60}$

5) $\sigma^2 = npq = 80(.75)(.25) = \boxed{15}$

6) $\sigma = \sqrt{\sigma^2} = \sqrt{15} \approx \boxed{4}$

7) $\underbrace{\text{Usual Range}}_{95\% \text{ Range}}$ $\mu \pm 2\sigma = 60 \pm 2(4) = 60 \pm 8 \Rightarrow \boxed{52 \text{ to } 68}$

8) P(between 52 and 68, inclusive, have a job)

$P(52 \leq x \leq 68) = \text{binomcdf}(80, .75, 68) - \text{binomcdf}(80, .75, 51)$

$\approx \boxed{.973}$

SG 16, Last Page → use exact value of p & q.

SG 16 ✓

$P = \frac{1}{6}$, $q = \frac{5}{6}$

Apr 10-9:41 AM

Working with Continuous random Variable

SG 14 – 16 Discrete

SG 17 - 20 Continuous

1) Uniform Prob. dist.

2) Standard normal Prob. dist.

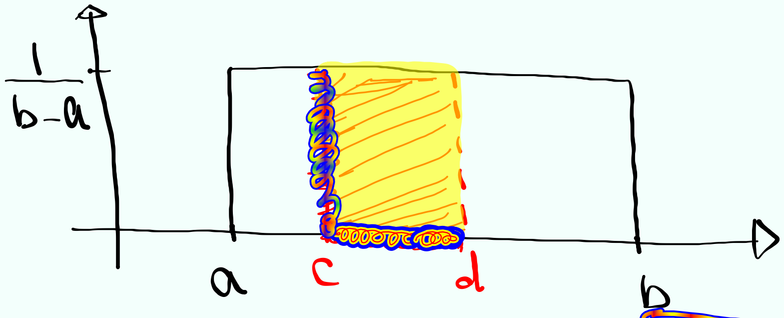
3) Normal Prob. dist.

4) Central limit Theorem

5) Applications

Apr 10-9:51 AM

Let x be a continuous random variable for all values from a to b with uniform Prob. dist.

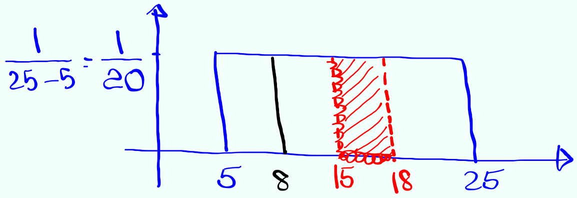


$P(x\underset{\uparrow\text{Line}}{=}c)=0$

$P(c<x<d)=\boxed{d-c}\cdot\boxed{\frac{1}{b-a}}$

Apr 10-9:54 AM

Consider a uniform Prob. dist for all values from 5 to 25

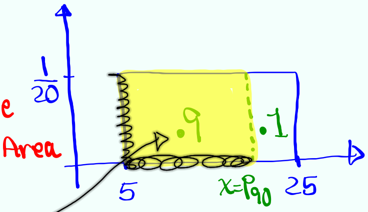


$P(x=8)=\boxed{0}$ $\quad$ $P(15<x<18)=(18-15)\cdot\frac{1}{20}=\boxed{\frac{3}{20}}$

Find $x=P_{90}$

90% below
Left Area
.9

10% above
Right Area
.1



$(x-5)\cdot\frac{1}{20}=.9$

$x-5=20(.9)$

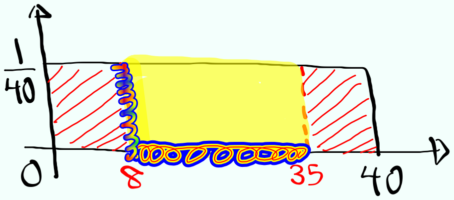
$x-5=18$

$\boxed{x=23}$

Apr 10-10:00 AM

Consider a uniform prob. dist. for all values from 0 to 40.

1) Draw & clearly label.



2) $P(x=5)=\boxed{0}$

3) $P(x<8 \text{ or } x>35)$

$=1-P(8<x<35)$

Total Prob.

$=1-(35-8)\cdot\frac{1}{40}$

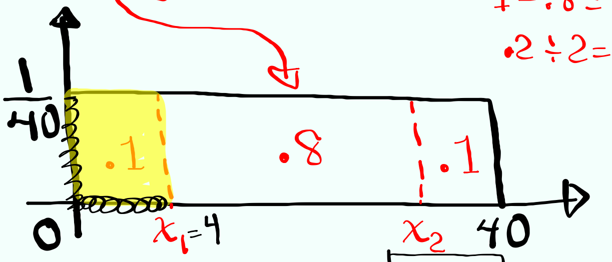
$=1-\frac{27}{40}=\boxed{\frac{13}{40}}$

Apr 10-10:08 AM

4) Find two values that separate the middle 80% from the rest.

$1 - .8 = .2$

$.2 \div 2 = .1$



$\boxed{x_2=36}$

$(x_1 - 0)\cdot\frac{1}{40} = .1$

$x_1 = 40(.1)$ $\boxed{x_1=4}$

SG 17
Pages 1 & 2

$(40-x_2)\cdot\frac{1}{40} = .1$

$40 - x_2 = 40(.1)$

$40 - x_2 = 4$

$40 - 4 = x_2$

$\boxed{x_2=36}$

Apr 10-10:14 AM

40 TKTS Sold

$25 each

one ticket randomly selected.

Winner gets iPad worth $500.

Find expected value per tkt sold.

40 TKT($25) = $1000

$1000 − $500 = $500 Net Profit

$500/40 TKts = $12.5

Net	P(Net)
25 − 500	1/40
25 − 0	39/40

Net → L1

P(Net) → L2

E.V. $= \mu = \bar{x} = \boxed{12.5}$

1-Var Stats
List: L1
FreqList: L2

$\sigma^2 = 6093.75$

$= \frac{24375}{4}$

Apr 10-10:20 AM

Pay me $5

Draw one tkt from a full deck of playing cards.

if you draw an Ace → I give you $25

" " " a face → " " " $5

Any other cards → " " " nothing

find expected value per play for the house.

Net	P(Net)	
5 −25	$\frac{4}{52}$	Ace
5 −5	$\frac{12}{52}$	face
5 −0	$\frac{36}{52}$	any other card.

Net → L1

P(Net) → L2

E.V. $= \mu = \bar{x}$

$\approx \$1.92$

$\sigma^2 = 44.37869822$

$= \frac{7500}{169}$

Apr 10-10:29 AM

7 Dimes

3 Quarters

take 2 Coins

No replacement

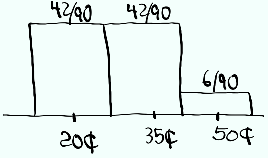
Sample Space

DD	DQ	QD	QQ
20¢	35¢		50¢

$P(20¢) = P(DD) = \frac{7}{10} \cdot \frac{6}{9} = \boxed{\frac{42}{90}}$

$P(35¢) = P(DQ \text{ or } QD) = 2 \cdot \frac{7}{10} \cdot \frac{3}{9} = \boxed{\frac{42}{90}}$

$P(50¢) = P(QQ) = \frac{3}{10} \cdot \frac{2}{9} = \boxed{\frac{6}{90}}$



Total	P(Total)
20¢	42/90
35¢	42/90
50¢	6/90

$\boxed{\mu = 29}$ $\boxed{\sigma = 9.165}$ $\boxed{\sigma^2 = 84}$

finish SG 15

" " 16

work on SG 17 (Pages 1 & 2).

Apr 10-10:37 AM